

V Semester B.A./B.Sc. Examination, November/December 2018
(CBCS) (2016 – 17 and Onwards) (Semester Scheme)
(Fresh + Repeaters)
MATHEMATICS – VI

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART – A

1. Answer **any five** questions.

(5×2 =10)

a) Write Euler's equation when f is independent of y .b) Find the differential equation of the functional $I = \int_{x_1}^{x_2} [y^2 - (y')^2 + 2ye^x] dx$.

c) Write the Euler's equation.

d) Evaluate $\int_C (3x + y)dx + (2y - x)dy$ along $y = x$ from $(0, 0)$ to $(10, 10)$.e) Evaluate $\int_0^{\pi/2} \int_0^{a \cos \theta} r^2 dr d\theta$.f) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz dx dy dz$.g) Find the area of the circle $x^2 + y^2 = a^2$ by double integration.

h) State Stoke's theorem.

PART – B

Answer **two full** questions.

(2×10 =20)

2. a) Find the extremal of the functional $I = \int_0^{\pi/2} [y^2 - (y')^2 - 2y \sin x] dx$ under the end conditions $y(0) = y(\pi/2) = 0$.

b) Define Geodesic. Prove that geodesic on a plane is a straight line.

OR

P.T.O.



3. a) If a cable hangs freely under gravity from two fixed points then show that the shape of cable is a catenary.

b) Solve the variational problem $\delta \int_0^{\pi/2} [y^2 - (y')^2] dx = 0$ under the condition

$$y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 2.$$

4. a) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

b) Find the extremal of the functional $\int_{x_1}^{x_2} [12xy + (y')^2] dx$.

OR

5. a) Find the extremal of the functional $\int_0^1 [x + y + (y')^2] dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$.

b) Find the extremal of the functional $\int_0^1 [(y')^2 + x^2] dx$ subject to the constraint $\int_0^1 y dx = 2$ and having end conditions $y(0) = 0$ and $y(1) = 1$.

PART - C

Answer two full questions.

(2×10=20)

6. a) Evaluate $\int_C (x + y + z) ds$ where C is line joining the points (1, 2, 3) and (4, 5, 6) whose equations are $x = 3t + 1$, $y = 3t + 2$; $z = 3t + 3$.

b) Evaluate $\iint_R xy(x + y) dx dy$ over the region R bounded between the parabola $y = x^2$ and the line $y = x$.

OR

7. a) Change the order of integration in $\int_0^{2\sqrt{ax}} \int_0^x x^2 dx dy$ and hence evaluate.

b) Evaluate $\iint_A \sqrt{4x^2 - y^2} dx dy$ where A is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.



8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx dy dz$

b) Change into polar coordinates and evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$

OR

9. a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration.

b) Evaluate $\iiint xyz \, dx dy dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing it to spherical polar coordinates.

PART - D

Answer two full questions.

(2×10 =20)

10. a) State and prove Green's theorem.

b) Using divergence theorem, evaluate $\iiint_S (x\hat{i} + y\hat{j} + z^2\hat{k}) \cdot \hat{n} \, ds$ where S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

OR

11. a) By using divergence theorem, evaluate $\iiint_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface enclosing the region for which $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$.

b) Evaluate $\iint \text{curl } \vec{F} \cdot \hat{n} \, ds$ by Stoke's theorem if $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.

12. a) Using Green's theorem evaluate for the scalar line integral of $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ over the rectangular region bounded by the lines $x = 0, y = 0; x = a; y = b$.



b) Using the divergence theorem evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$ where

$\vec{F} = (x^2 - yz) \hat{i} + (y^2 - zx) \hat{j} + (z^2 - xy) \hat{k}$ over the rectangular parallelepiped

$$0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c.$$

OR

13. a) Using Green's theorem evaluate $\int_C (xy + y^2) dx + x^2 dy$ where C is the closed curve bounded by $y = x$ and $y = x^2$.

b) Evaluate by Stoke's theorem $\oint_C yz dx + zx dy + xy dz$. where C is the curve $x^2 + y^2 = 1; z = y^2$.
